Objectives:

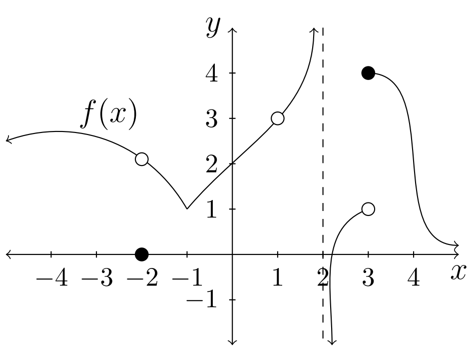
- Define continuity of a function (from the left, from the right, at a point, and over its domain)
- Determine if a function is continuous at a point or on its domain

Intuition: A function is continuous if you can draw its graph without lifting your pencil. This means it has no ____ .

Definitions:

- **CONTINUOUS:** A function $f(x)$ is continuous at a number a if

Graphical Example:



This graph is discontinuous at

(a) $x =$ _______

(b) $x =$ _______

(c) $x =$ _______

(d) $x =$ _______

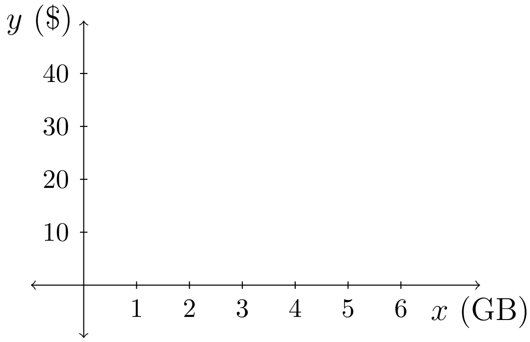
There are three requirements hidden in this definition:

If $\lim_{x \to a} f(x)$ exists but isn't equal to $f(a)$, we call $x = a$ a ________________________ .
There are two other types of discontinuities: jumps and vertical asymptotes.

- **CONTINUOUS FROM THE RIGHT:** A function $f(x)$ is continuous from the right at a number a if

- **CONTINUOUS FROM THE LEFT:** A function $f(x)$ is continuous from the left at a if

Example: Let $p(x)$ be the price I pay for data on my cell phone plan as a function of the number of GB I purchase. If I buy 2GB or less, I pay \$20. If I buy more than 2GB but no more than 4GB, I pay \$30. If I purchase more than 4GB, I pay \$35. If I don't purchase any data plan, I don't pay anything.



$$p(x) = \begin{cases} 0 & x = 0 \\ 20 & 0 < x \leq 2 \\ 30 & 2 < x \leq 4 \\ 35 & x > 4 \end{cases}$$

The function $p(x)$ is discontinuous at ______________________________ . The function is left continuous but not continuous at ____________________ .

Question: Which functions are continuous? To answer this question, we need to think back to the direct substitution property which gives us that polynomials and rational functions satisfy

$$\lim_{x \to a} f(x) = f(a).$$

This means polynomial and rational functions are ____________________ !

Conclusion: The following functions are continuous on their domains:

Example: Where is $f(x) = \dfrac{1}{\sqrt{5 - 3x}}$ continuous?